ANALYSIS OF THE MOTION OF A SUBSONIC VAPOR FLOW IN A QUASI-CLOSED VOLUME

Yu. Z. Bubnov and M. N. Libenson

UDC 532.529.5

The motion of a vapor is described for the case in which the influence of geometry is isolated. The parameters are determined for a subsonic vapor flow on the outer boundary of a gas-kinetic layer near a surface of vaporization. Profiles of the gas-dynamic variables as a function of the dimensionless coordinate are calculated.

The authors and others [1, 2] have analyzed the motion of a vapor in a quasi-closed volume for the case in which the influence of flow rate is isolated. The kinetic relations obtained in [3, 4] between the initial gas-dynamic parameters of the vapor and the temperature of the vaporization surface were tested experimentally and the profiles determined for the variation of the gas-dynamic parameters of the vapor during transit in a cylindrical chamber with condensation and revaporization at the walls.

The objective of the present study is to analyze the motion of a vapor in a closed volume* for the case in which the influence of geometry is isolated.

We consider the case of vaporization in a truncated convergent cone with base diameters D_0 and D_L ($D_0 > D_L$) and height L with the entire volume at one given temperature ($dT_c/dx = 0$) close to the vaporization point, except for the force-cooled chamber cover, where the vapor is completely condensed. In this case there is no net vapor condensation on the side walls of the chamber [1, 2], and the variation of the vapor parameters as a function of the coordinate x is determined solely by the geometry of the problem.†

The equation for the Mach number $M = M(F)$ in the case of isolated geometrical influence has the form [5]

$$(M^2-1)\frac{dM^2}{M^2} = 2\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dF}{F}. \tag{1}$$

For an arbitrary chamber cross section $D_x = D_0 - 2x\cot\varphi$

$$\frac{dF_x}{F_x} = -\frac{4\cot\varphi}{D_0-2x\cot\varphi}\,dx. \tag{2}$$

Substituting (2) into (1) and separating variables, we obtain

$$\frac{M^2-1}{M\left(1+\frac{\gamma-1}{2}M^2\right)}\,dM = -\frac{2}{\frac{D_0}{2\cot\varphi}-x}\,dx. \tag{3}$$

We reduce Eq. (3) to dimensionless form. The boundary cross section of the gas-dynamic problem is a certain cross section x_1 situated at a distance of (2 to 3) λ from the vaporization surface [3]. To simplify the problem we assume that $x_1 \simeq 0$, which is sufficiently rigorous, because $L \gg \lambda$ for $P_S(T_0) > 10^{-2}$ torr.

*The chamber construction is described in detail in [1].

†The variation of the heat balance of the moving vapor due to condensation and revaporization, as in [1, 2], can be neglected.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 3, pp. 460-466, September, 1973. Original article submitted March 28, 1973.

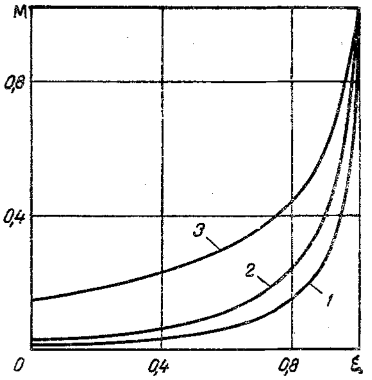

Fig. 1

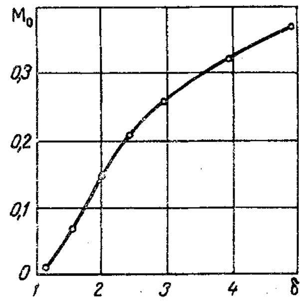

Fig. 2

Fig. 1. Curves of M = M(ξ) for different values of the parameter δ: 1) δ = 1.2; 2) 1.33; 3) 2.

Fig. 2. Mach number in the initial gas-dynamic cross section versus geometrical parameters of the chamber.

Then

$$\frac{M^2-1}{M\left(1+\frac{\gamma-1}{2}M^2\right)}\,dM=-\frac{2}{\delta-\xi}\,d\xi, \tag{4}$$

where

$$\xi=\frac{x}{L};\quad \delta=\frac{D_0}{2L\cot\varphi}.$$

By a transformation of the left-hand side of Eq. (4) we obtain

$$\frac{\frac{2}{\gamma-1}}{M}\,dM-\frac{\frac{\gamma+1}{\gamma-1}}{M\left(1+\frac{\gamma-1}{2}M^2\right)}=-\frac{2}{\delta-\xi}\,d\xi. \tag{5}$$

Integrating, we obtain

$$\frac{1}{M}\left(\frac{2}{\gamma-1}+M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}=C(\delta-\xi)^2. \tag{6}$$

We evaluate the constant of integration from the condition M = 1 at the critical gas-dynamic cross section. For an isolated geometrical influence the critical cross section is the minimum cross section [6], i.e., in our problem the cross section* F_L.

Thus, M = 1 for ξ = 1, so that

$$C=\frac{\left(\frac{1+\gamma}{\gamma-1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{(\delta-1)^2}$$

and the equation for M = M(ξ) has the form

$$\frac{1}{M}\left(\frac{2}{\gamma-1}+M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}=\frac{\left(\frac{1+\gamma}{\gamma-1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{(\delta-1)^2}(\delta-\xi)^2. \tag{7}$$

For a diatomic vapor (γ = 7/5)

*It is essential to note that high-frequency disturbances can in principle propagate from the cross section F_L downstream in the subsonic vapor, inducing an instability of the investigated flow regime. However, if a cooled cover is placed in the cross section F_L, where the condensate can only accumulate, practically without revaporization, the subsonic flow becomes stable.

$$\frac{1}{M}(5+M^2)^3=\frac{216}{(\delta-1)^2}(\delta-\xi)^2. \tag{8}$$

Solving Eq. (8) for ξ, we obtain

$$\xi=\delta-\frac{\delta-1}{14{,}7\sqrt{M}}\sqrt{(5+M^2)^3}. \tag{9}$$

Curves of $M = M(\xi)$ plotted according to Eq. (9) for various values of δ are given in Fig. 1. It is apparent from the figure that as $\xi \to 0$ the Mach number becomes considerably less than unity, and for $\xi = 0$ Eq. (9) can be solved explicitly for δ:

$$M_0\simeq 0{,}58\left(\frac{\delta-1}{\delta}\right)^2. \tag{10}$$

The dependence $M_0 = M_0(\delta)$ is given in Fig. 2. To find the profiles of the other gas-dynamic variables we need to evaluate them in the initial gas-dynamic cross section as a function of M_0, i.e., find $n_1 = f(M_0)$ and $T_1 = f(M_0)$.

We make use of the solution of the gas-kinetic problem for the determination of the relation of n_1 and T_1 with n_0 and T_0 with regard for the fact that in our case the vapor flows at a subsonic velocity, i.e., $u_1 < c_1(T_1)$, and we write the following initial expressions:

$$n_1u_1=n_0\sqrt{\frac{kT_0}{2\pi m}}-\beta\frac{n_1\sqrt{kT_1}}{\sqrt{2m}}\,i\Phi^*\left(u_1\sqrt{\frac{m}{2kT_1}}\right), \tag{11a}$$

$$n_1u_1^2+\frac{n_1kT_1}{m}=\frac{n_0kT_0}{2m}+\frac{2\beta kT_1n_1}{m}\,i^2\Phi^*\left(u_1\sqrt{\frac{m}{2kT_1}}\right), \tag{11b}$$

$$n_1u_1\left(\frac{5}{2}kT_1+\frac{mu_1^2}{2}\right)=\frac{mn_0}{2\sqrt{\pi}}\left(\frac{2kT_0}{m}\right)^{3/2}-\frac{mn_1\beta}{4}$$
$$\times\left(\frac{2kT_1}{m}\right)^{3/2}\left[i\Phi^*\left(u_1\sqrt{\frac{m}{2kT_1}}\right)+6i^3\Phi^*\left(u_1\sqrt{\frac{m}{2kT_1}}\right)\right], \tag{11c}$$

where β is an unknown quantity characterizing the fraction of reverse particle flux from infinity to the vaporization surface and $i^n\Phi^*(z)$ are the error integrals [7].

Recognizing that $c_1 = \sqrt{\gamma(kT_1/m)}$, we transform the argument of the function $i^n\Phi^*$ to the form

$$u_1\sqrt{\frac{m}{2kT_1}}=\frac{u_1}{c_1}\sqrt{\frac{\gamma kT_1m}{2mkT_1}}=M_0\sqrt{\frac{\gamma}{2}}. \tag{12}$$

We now introduce the new variable $\theta = M_0\sqrt{\gamma/2}$, where $M_0 = u_1/c_1$. Then

$$n_0\sqrt{\frac{kT_0}{2\pi m}}=n_1u_1\left[1+\frac{\beta}{2\theta}\,i\Phi^*(\theta)\right], \tag{13a}$$

$$\frac{n_0kT_0}{2m}=n_1u_1^2\left[1+\frac{1}{2\theta^2}-\frac{\beta}{\theta^2}\,i^2\Phi^*(\theta)\right], \tag{13b}$$

$$n_0\left(\frac{2kT_0}{m}\right)^{3/2}=n_1u_1^3\sqrt{\pi}\left\{\frac{5}{2\theta^2}+1+\frac{\beta}{2\theta^3}\left[i\Phi^*(\theta)+6i^3\Phi^*(\theta)\right]\right\} \tag{13c}$$

We denote

$$f_1(\beta,\theta)=1+\frac{\beta}{2\theta}\,i\Phi^*(\theta), \tag{14a}$$

$$f_2(\beta,\theta)=1+\frac{1}{2\theta^2}-\frac{\beta}{\theta^2}\,i^2\Phi^*(\theta), \tag{14b}$$

$$f_3(\beta,\theta)=1+\frac{5}{2\theta^2}+\frac{\beta}{2\theta^3}\left[i\Phi^*(\theta)+6i^3\Phi^*(\theta)\right]. \tag{14c}$$

Then from (13a) and (14b) we have

$$n_1=\frac{n_0}{\pi}\,\frac{f_2(\beta,\theta)}{f_1^2(\beta,\theta)}, \tag{15}$$

and from (13b), taking (15) into account,

$$T_1 = \frac{\pi}{4\theta^2}\left[\frac{f_1(\beta,\ \theta)}{f_2(\beta,\ \theta)}\right]^2 T_0. \tag{16}$$

Multiplying (13a) by (13b) and dividing by (13c), we obtain another relation between n_1 and n_0:

$$n_1 = \frac{n_0}{8}\,\frac{f_3(\beta,\ \theta)}{f_1(\beta,\ \theta)\cdot f_2(\beta,\ \theta)}\,. \tag{17}$$

From (15) and (17) we obtain the following algebraic equation for the determination of β:

$$\frac{8}{\pi} f_2^2(\beta,\ \theta) = f_1(\beta,\ \theta)\, f_3(\beta,\ \theta). \tag{18}$$

Equation (18) represents a quadratic equation in β:

$$\frac{8}{\pi}(b^2 - 2bc\beta + c^2\beta^2) = d + ad\beta + g\beta + ag\beta^2,$$

in which a, b, c, d, and g are determined from (14a)-(14c) as follows:

$$f_1(\beta,\ \theta) = 1 + a\beta;\ \ f_2(\beta,\ \theta) = b - c\beta;\ \ f_3(\beta,\ \theta) = d + g\beta;$$

$$a = \frac{i\Phi^*(\theta)}{2\theta};\ b = 1 + \frac{1}{2\theta^2};\quad c = \frac{i^2\Phi^*(\theta)}{\theta^2};$$

$$d = 1 + \frac{5}{2\theta^2};\quad g = \frac{i\Phi^*(\theta) + 6i^3\Phi^*(\theta)}{2\theta^3}.$$

The solution of Eq. (18) subject to the adopted notation has the form

$$\beta = \frac{\frac{16}{\pi} bc + ad + g \pm \sqrt{\left(\frac{16}{\pi} bc + ad + g\right)^2 - 4\left(\frac{8}{\pi} c^2 - ag\right)\left(\frac{8}{\pi} b^2 - d\right)}}{2\left(\frac{8}{\pi} c^2 - ag\right)}. \tag{19}$$

The results of a numerical solution of (19) are given in Fig. 3. The solutions of Eq. (16) and (17) are given in the figure in the form $T_1/T_0 = f(\theta)$ and $n_1/n_0 = f(\theta)$. It follows from the solution $M_0 = f(\delta)$ (Fig. 2) that M_0 is considerably less than unity over a wide range of variation of the parameter δ. Accordingly, we consider separately the case of small M_0 and, hence, small θ. It is evident from Fig. 3 that for $\theta \leqslant 0.4$ we have $\beta \simeq 1$ and $T_1/T_0 \simeq 1$. This situation corresponds physically to the condition when the kinetic energy of the directional subsonic vapor flow is much less than its internal energy, i.e.,

$$\frac{mu_1^2}{2} \ll \frac{3}{2} kT_1.$$

We now find an analytical expression for $n_1/n_0 = f(\theta)$ under the given assumptions. According to (16) $f_2(\theta)/f_1(\theta) \simeq \sqrt{\pi/2\theta}$. Then on the basis of (15)

$$\frac{n_1}{n_0} = \frac{1}{\pi}\,\frac{f_2(\beta,\ \theta)}{f_1^2(\beta,\ \theta)} \simeq \frac{1}{2\sqrt{\pi}\,\theta f_1(\theta)}\,. \tag{20}$$

For a more precise calculation of the ratio $n_1/n_0 = f(\theta)$ we approximate $\beta = f(\theta)$ in the interval $0 \leqslant \theta \leqslant 0.4$ by the linear function $\beta = 1 + \alpha\theta$. It is seen in Fig. 3 that the line $\beta = 1 + 1.8\theta$ practically coincides in the indicated interval with the curve plotted for $\beta = f(\theta)$ according to Eq. (19). Then, taking (14a) and (14b) into account, we have

$$\frac{n_1}{n_0} \simeq \frac{1}{2\sqrt{\pi}\theta\left[1 + \frac{\beta}{2}\left(\frac{1}{\sqrt{\pi}\theta} - 1\right)\right]}$$

$$= \frac{1}{2\sqrt{\pi}\theta\left(\frac{1}{2} + \frac{1}{2\sqrt{\pi}\,\theta} + \frac{\alpha}{2\sqrt{\pi}} - \frac{\alpha}{2}\,\theta\right)}$$

or, neglecting the term $(\alpha/2)\theta$,

$$\frac{n_1}{n_0} \simeq \frac{1}{1 + (\sqrt{\pi} + \alpha)\,\theta}\,.$$

Finally,

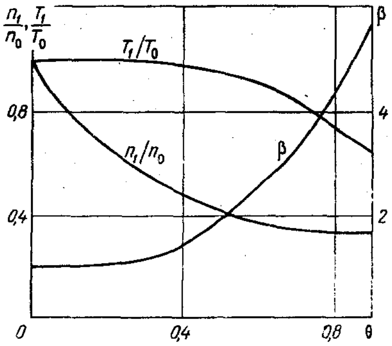

Fig. 3

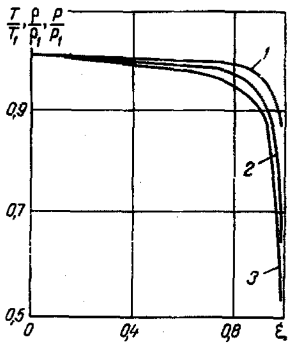

Fig. 4

Fig. 3. Parameters of vapor in the initial gas-dynamic cross section versus Mach number.

Fig. 4. Normalized curves of vapor parameters versus dimensionless coordinate. 1) $T/T_1 = f(\xi)$; 2) $\rho/\rho_1 = f(\xi)$; 3) $P/P_1 = f(\xi)$.

$$\frac{n_1}{n_0} \simeq \frac{1}{1+2.5\theta} = \frac{1}{1+2.5\sqrt{\frac{\gamma}{2}}M_0}. \tag{21}$$

The expression thus obtained yields good agreement with the exact values of n_1/n_0 calculated according to Eq. (17), up to $\theta \simeq 0.4$, and can be used to determine the vapor density in the initial gas-dynamic cross section. In the derivation of the gas-dynamic relations we used a technique developed in [3] and based on the approximation of the particle distribution function in a Knudsen vapor layer by the Tamm—Mott—Smith bimodal function. Here, as opposed to [3], we use this approach for an arbitrary value of the Mach number ($M < 1$) at the outer boundary of the gas-kinetic layer, rather than for $M = 1$ only. The legitimacy of this approximation is upheld insofar as the thickness of the Knudsen layer is small in the event of an abrupt change of the distribution function at its boundaries (namely ~2 or 3 particle mean free paths). The resulting approximate gas-kinetic relations shown in Fig. 3 are in good agreement with the exact solution given for the gas-kinetic problem in [8].

Knowing the values of the vapor parameters in the initial gas-dynamic cross section, we can now use the well-known expressions of [6] to calculate the profiles of the density, pressure, temperature, and other gas-dynamic variables as a function of the coordinate, because they are all uniquely expressed in terms of M and the initial vapor parameters n_1 and T_1.

Normalized curves of the vapor parameters $\rho/\rho_1 = f(\xi)$; $p/p_1 = f(\xi)$; $T/T_1 = f(\xi)$ as a function of the dimensionless coordinate ξ are given in Fig. 4 for the above-stated conditions.

The foregoing theoretical analysis of the motion of a subsonic vapor flow in a conical chamber therefore shows that the vapor remains hot ($T_1 \simeq T_0$) and dense over a wide range of variation of the chamber geometrical parameters and vaporization temperature, even at a large distance from the vaporization surface. Consequently, a quasi-closed volume having a conical profile is well recommended for the production of vacuum condensates with a thermodynamic-equilibrium crystalline structure, where the substrates can be oriented either perpendicular or parallel to the vapor flow.

NOTATION

x is the coordinate measured from the vaporization surface;
F is the cross-sectional area of the chamber;
T_0 is the temperature of the vaporization surface;
T_W is the chamber wall temperature;
T is the vapor temperature;
P is the vapor pressure;
ρ is the vapor density;
M is the Mach number;
P_S is the saturated vapor pressure;

λ	is the mean free path of the vapor molecules;
φ	is the angle between the large base and generatrices of the chamber;
u_1, T_1, ρ_1, P_1	are the velocity, temperature, density, and pressure of the vapor in the initial gas-dynamic cross section.

LITERATURE CITED

1. Yu. Z. Bubnov, M. N. Libenson, M. S. Lur'e, and G. A. Filaretov, Inzh.-Fiz. Zh., 17, No. 3 (1969).
2. Yu. Z. Bubnov, M. N. Libenson, M. S. Lur'e, V. S. Ravin, and G. A. Filaretov, Inzh.-Fiz. Zh., 21, No. 4 (1971).
3. S. I. Anisimov, Zh. Éksp. Teor. Fiz., 54, No. 1, 339 (1968).
4. G. S. Romanov and V. K. Pustovalov, Izd. Akad. Nauk Beloruss. SSR, Ser. Fiz., 28, No. 3 (1968).
5. P. A. Vulis, Thermodynamics of Gas Flows [in Russian], GÉI, Moscow—Leningrad (1950).
6. G. N. Abramovich, Applied Gas Dynamics [in Russian], GITTL, Moscow (1953).
7. I. N. Bronshtein and K. N. Semendyaev, Handbook of Mathematics [in Russian], GITTL, Moscow (1957).
8. M. M. Kogan and N. K. Makashev, Izv. Akad. Nauk SSSR, Mekhan. Zhidk. i Gaza, No. 6, 3 (1971).